

A Review of Ising Machines Implemented in Conventional and Emerging Technologies

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Abstract—Ising machines have received growing interest as efficient and hardware-friendly solvers for combinatorial optimization problems (COPs). They search for the absolute or approximate ground states of the Ising model with a proper annealing process. In contrast to Ising machines built with superconductive or optical circuits, complementary metal–oxide–semiconductor (CMOS) Ising machines offer inexpensive fabrication, high scalability, and easy integration with mainstream semiconductor chips. As low-energy and CMOS-compatible emerging technologies, spintronics and phase-transition devices offer functionalities that can enhance the scalability and sampling performance of Ising machines. In this article, we survey various approaches in the process flow for solving COPs using CMOS, hybrid CMOS-spintronic, and phase-transition devices. First, the methods for formulating COPs as Ising problems and embedding Ising formulations to the topology of the Ising machine are reviewed. Then, Ising machines are classified by their underlying operational principles and reviewed from a perspective of hardware implementation. CMOS solutions are advantageous with denser connectivity, whereas hybrid CMOS-spintronic and phase-transition device-based solutions show great potential in energy efficiency and high performance. Finally, the challenges and prospects are discussed for the Ising formulation, embedding process, and implementation of Ising machines.

Index Terms—Ising machines, spintronics, phase-transition devices, oscillator, annealing, combinatorial optimization.

I. INTRODUCTION

The Ising model was initially proposed for the theoretical description of ferromagnetism. Ernst Ising then solved a one-dimensional model consisting of a linear chain of spins interacting only with their nearest neighbors [1]. It did not get the attention of many physicists until Lars Onsager analytically solved a two-dimensional Ising model and discovered the phase transition phenomenon [2]. The magnetism disappears when a magnet is heated above a certain temperature and

appears when cooled below it, which is called a phase transition. Kenneth Wilson further obtained the values of critical exponents to describe how different properties of a substance change during a phase transition. It was then shown that a series of unrelated substances share the same critical exponents [3]. This critical phenomenon exists in many systems, such as gas-liquid phase transition processes, turbulent flow, stock markets, and economic systems. Two important features of the system are the scale-invariance and persistent relationships over time. This universality indicates that a system composed of many interacting individuals, described by a pair of antonyms such as “up-down” and “with-without”, can be studied based on the Ising model.

The statistical mechanics of the Ising model have been explored for solving combinatorial optimization problems (COPs) since the 1980s [4], [5]. Finding the ground state of the Ising model is nondeterministic polynomial-time (NP)-hard. For this reason, new computing models and algorithms are being studied for pursuing further performance improvement in the post-Moore era. In particular, domain-specific architectures that take advantage of the hardware-friendly nature of an Ising model-based solver, as known as Ising machines, have recently garnered significant attention [6], [7]. Some concepts from neural networks can be also applied to Ising machines to increase their performance [8].

A COP is solved by an Ising machine, as illustrated in Fig. 1. A real-world problem is first formulated as an Ising problem. The Ising formulation based on a logical Ising model (LIM) is then embedded into the topology of a physical Ising model (PIM). Subsequently, the Ising machine searches for suboptimal/optimal solutions. Finally, the obtained result is interpreted as a solution for the original problem. Ising machines are classified into three types, based on the underlying technologies:

- A quantum Ising machine implements the spins by using superconducting flux quantum bits (called qubits) based on quantum annealing. It achieves an ultra-fast search for solutions due to the use of quantum fluctuations. A machine with 128 qubits, called the D-Wave quantum annealer, was first released in 2011 [6]. However, coherence must be guaranteed to maintain the quantum superposition states of qubits [9]. Therefore, it limits the connections between qubits. Recently, an advanced machine with 5k+ qubits with an increased connectivity of each qubit to fifteen has been made

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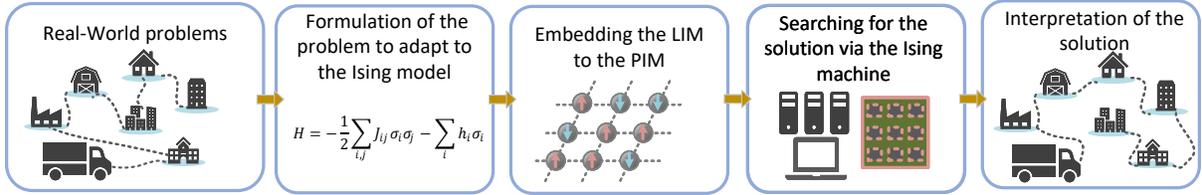


Fig. 1. Solving a combinatorial optimization problem using an Ising machine.

possible [9]. However, due to technology limitations, a large physical space and relatively high power are required to ensure an ultra-low operating temperature [10], [11].

- With high scalability and flexible connectivity, an optical Ising machine uses optical pulses as spins, which operates at room temperature. The matrix-vector multiplication is implemented using a Field Programmable Gate Array (FPGA). Honjo et al. reported an optical Ising machine with more than 100k spins and 10 billion spin-spin connections [12]. However, using hundreds of meters of optical fibre cavities leads to stability issues and high fabrication costs; relying on electronics also restricts the optical system's efficiency. This limitation presents a challenge for system miniaturization and large-scale integration. To deal with it, some efforts attempt to develop systems based on coupled lasers [13], optoelectronics [14], and exciton-polaritons [15], but with a sacrifice in scalability and spin connectivity.
- Complementary metal oxide semiconductor (CMOS) and CMOS compatible Ising machines emulate physical phenomena based on heuristic algorithms [16] or implement a dynamical system using hardware as classical spins [17]. Although the speed and scalability are not competitive to quantum or optical solutions, CMOS [18] or CMOS compatible circuits [19] provide higher reliability, which present a potential industrial solution for building Ising machines in the near future.

This article reviews the state-of-the-art studies throughout the design process flow of Ising machines using conventional and emerging technologies, in particular, CMOS and CMOS-compatible ones, as shown in Fig. 1. First, we introduce the quadratic unconstrained binary optimization (QUBO) formulation for a COP to be fit into a LIM; approaches of converting COPs represented by non-binary variables and higher-order polynomials to the QUBO are then presented. An Ising formulation needs to adapt to the topology of the PIM and satisfy its bit width requirement. Therefore, the embedding methods for transforming the topology and coefficients in a LIM to fit in a PIM are illustrated. Subsequently, CMOS and CMOS compatible circuit-based Ising machines are reviewed from the perspectives of underlying operation principles and circuit design.

This article is organized as follows. Sections II and III present the Ising formulations of COPs and embedding methods, respectively. CMOS and CMOS compatible Ising machines are classified and introduced in Sections IV and V. Section VI discusses existing implementations and emerging applications. Section VII concludes this article and discusses future challenges and prospects.

II. ISING FORMULATION OF COMBINATORIAL OPTIMIZATION

The Ising model mathematically simulates the ferromagnetism in an array of magnetic spins. The Hamiltonian H of the Ising model with external magnetic fields is given by [20]

$$H(\boldsymbol{\sigma}) = -\frac{1}{2} \sum_{i,j} J_{ij} \sigma_i \sigma_j - \sum_i h_i \sigma_i, \quad (1)$$

where $\boldsymbol{\sigma}$ is a vector of spin variables, σ_i (or σ_j) denotes the state of the i th (or j)th spin that takes the value -1 (as the downward state) or $+1$ (as the upward state), J_{ij} is the interaction coefficient for σ_i and σ_j , and h_i is the external magnetic field on σ_i , as shown in Fig. 2. Although higher order Ising models have recently been studied [21], we focus on the most-widely considered model of (1) in this article.

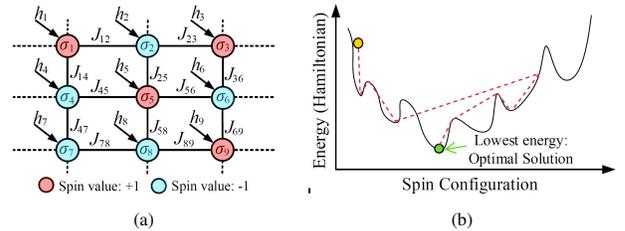


Fig. 2. The Ising model: (a) A 9-spin Ising model with 2D lattice topology, and (b) An energy landscape.

To convert a COP to a problem of energy minimization of the Ising model in (1), the first step is to formulate the COP to a QUBO problem using linear and quadratic terms. Let $\mathbb{B} = \{0, 1\}$ be a binary set and \mathbb{R} be a real number set. A QUBO problem is described by [22]

$$f(\mathbf{x}) = \sum_{i,j} a_{ij} x_i x_j + \sum_i b_i x_i + c, \quad (2)$$

where \mathbf{x} is a vector of binary variables, x_i (or $x_j \in \mathbb{B}$) is an element in \mathbf{x} , $a_{ij} \in \mathbb{R}$, $b_i \in \mathbb{R}$ and $c \in \mathbb{R}$ denote the weight matrix element, the weight vector element and a constant scalar, respectively. The constant scalar is usually disregarded.

The second step is to convert the QUBO formulation in (2) to an Ising formulation using spin variables. Let $\mathbf{x} = \frac{1+\boldsymbol{\sigma}}{2}$, where \mathbf{x} is the vector of binary variable in (2) and $\boldsymbol{\sigma}$ is a vector of spin variables in (1). The QUBO formulation can be easily mapped to the energy function of an Ising model in (1) by considering [23]

$$h_i = -\left(\frac{b_i}{2} + \sum_j \frac{a_{ij}}{2}\right), \quad (3)$$

$$J_{ij} = -\frac{a_{ij}}{2}. \quad (4)$$

Lucas discussed the Ising formulations for COPs in detail [24]. An open-source Python library for constructing QUBOs from the objective functions has been developed [25].

COPs can be categorized into three types [23]: (1) those without constraints, such as the max-cut problem (MCP) and the number partitioning problem (NPP); (2) those with equality constraints, such as the traveling salesman problem (TSP), the quadratic assignment problem (QAP), the maze problem (MAP), and the graph partitioning problem (GPP); and (3) those with inequality constraints, such as the knapsack problem (KP) and the maximum satisfiability (MAX-SAT). The first class of COPs is formulated by using only a cost function, i.e., a minimization or maximization function. For the other two classes, a common way is to first describe the constraints as minimization functions, which are then added to the objective minimization function to construct the cost function for the QUBO formulation. Some parameters are introduced to balance the relative weight between the objective function and the constraints, which can be optimized for different Ising machines.

There are two challenges for the Ising formulation. One is how to smooth the energy landscape of the Ising model to decrease the probability of being stuck at the local minimum. Shirai et al. reduced the number of spin variables by merging several into one, thus simplifying the energy landscape [26]. However, the set of merged variables must be carefully selected for each iteration since the merging process might change the optimal solution with the lowest Ising energy. Ohno et al. considered auxiliary constraint conditions in the objective function that the optimal solution must satisfy. It reduces the search space and thus decreases the number of local minima in the energy landscape, although at a cost of time to extract the valid configuration of spin variables [27].

The other challenge is how to efficiently convert higher-order polynomials to second-order terms. Many COPs are expressed in general forms of higher-order polynomials with terms that contain products of more than two binary variables. Therefore, to fit in the Ising model formulated by using a second-order polynomial, a third step, called quadratization, is additionally required for reducing higher-order terms in the polynomial to second-order. There are a variety of quadratization techniques to convert higher-order polynomials [28], referred to as polynomial unconstrained binary optimization, to Ising formulation. A natural way is to transform k -th order interactions into quadratic ones by introducing arbitrary variables based on Rosenberg's polynomial [29]. However, it will incur a polynomial increase in the number of variables leading to an undesirable scaling of the problem size [30], [31]. Therefore, Mandal et al. proposed two compressed quadratization algorithms for sparse higher-order Ising problems [32]. For the COPs expressed by integer variables, there are three typical binary-integer encoding methods: one-hot, binary, and unary encoding [33]. Unary encoding can find a better solution for large KPs [34]. However, questions such as whether the performance is independent of the type of COPs and the choice of the Ising machine need further investigation.

III. EMBEDDING A LOGICAL ISING MODEL (LIM) TO A PHYSICAL ISING MODEL (PIM)

In the Ising formulation, a COP is described by and formulated to a LIM. In a LIM, any pair of spins can interact with

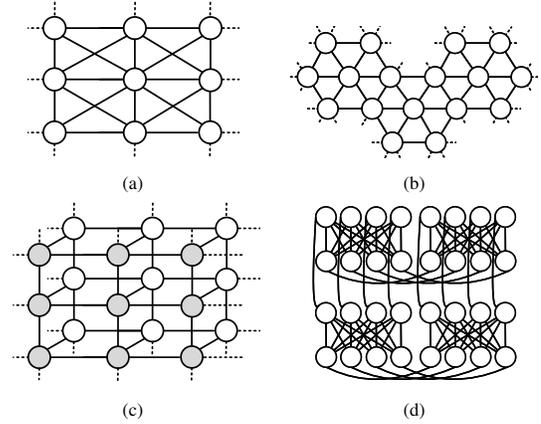


Fig. 3. Four types of topologies. (a) King's graph [35], (b) Hexagonal graph [36], (c) 3D lattice [37], and (d) Chimera graph [38].

each other [39]. However, the limitations in the physical implementation of the Ising machine are not considered, such as the topology and representation precision of the coupling coefficients. In a PIM, interactions exist between pairs of spins that are physically connected on the topology of a particular Ising machine [39] and the numerical range for the coupling coefficients is limited. There are different types of topologies. Fig. 2(a) shows the simple 2D lattice topology and Fig. 3 presents four other typical types. For example, given an Ising machine with the King's graph topology and 4-bit coupling coefficients, a COP formulated as a fully connected LIM with double precision coefficients cannot be naturally implemented in a PIM with a sparsely connected topology and a reduced precision. Therefore, topology and coefficient transformations are required to adapt a LIM to the specific topology and numerical range in a PIM [40].

A. Topology Transformation

A technique called minor embedding (ME) has been considered for topology transformation [41]. It maps a spin in the LIM to a set of spins, called a chain in the PIM. The spins in the same chain are ferromagnetically coupled and in an identical state. An ME process first identifies whether the LIM can be minor-embedded, then schedules spin operations with many replicas, and finally, decides the best-suited chain strength coupling [42].

Finding a minor embedding of the LIM into the PIM is an NP-hard problem, which requires an exponentially increasing runtime with the number of spins via exhaustive search [43]–[45]. Therefore, various heuristic methods have recently been developed for efficient ME. A well-known MinorMiner algorithm repeatedly attempts to find a valid embedding from an arbitrary LIM to an arbitrary PIM [42]. However, it is highly dependent on the initial selection of chains [42]. Multiple trials can mitigate this problem but at a cost of increased runtime. Moreover, the computational time and the number of auxiliary spins for constructing chains drastically increase when embedding a dense LIM to a PIM with a sparse topology. To reduce the search space, improved algorithms have been developed for specific topology of LIMs and PIMs [46]–[48]. To save the embedding overhead, Ohno et al. attempted to reduce the

quadratic terms in an Ising formulation to make the connectivity between spins less dense by introducing auxiliary penalties [27]. However, more constraints increase the complexity of the post-processing to verify whether the solution found satisfies the hard constraints.

B. Coefficient Transformation

A straightforward approach for coefficient transformation is to truncate the less significant bits by performing right bit-shift operations [40], referred to as the shift method. This method is easy to implement, but it will likely change the ground state of the original LIM, thus compromising the quality of solving the targeted COP. The spin-adding method [40] introduces auxiliary spins to convert the interaction and external magnetic field coefficients in two separate steps, which guarantees the exact mapping of the ground state of the LIM to that of the PIM. However, the number of the introduced auxiliary spins exponentially increases with the reduction of the bit-width of the coefficients. Yachi et al. used both the shift method and spin-adding method [49]. The former is employed until the absolute values of coefficients fall below a specified threshold, followed by the application of the latter method. This approach uses a moderate number of redundant spins without a significant change of the Hamiltonian in the LIM.

IV. CMOS DIGITAL AND ANALOG ISING MACHINES

A. CMOS Digital Ising Machines

CMOS digital Ising machines are developed by using different heuristic algorithms to carry out Monte Carlo (MC) simulations for the Ising model.

1) Classical Annealing Ising Machines

Classical annealing Ising machines emulate the thermal annealing process in metallurgy using importance sampling-based MC simulations [4]. At the beginning, the spin states are initialized randomly at a high temperature. Subsequently, as the temperature drops, the Hamiltonian decreases by flipping the spin states, and the probability of spin flips decreases as well. When the temperature becomes sufficiently low, the Ising model converges into the (near-)ground state. The major components are the annealing controller and the spin operator. The annealing controller computes the temperature for annealing and generates random numbers for computing the flip probability of the spin states. The spin operator determines the new spin states depending on the current states, the temperature, and random numbers. In what follows, existing annealing Ising machines are broadly classified into three categories, as shown in Fig. 4.

As shown in Fig. 4(a), CMOS annealing (CMOSA) examines only one spin at a time and then updates it randomly [35], [50], [52]. The states of disconnected spins are updated in each iteration (i.e., one MC simulation step). Due to the sequential update, CMOSA machines usually implement sparsely connected topologies.

Digital annealing (DA) applies a parallel-trial scheme to increase the spin flip probability to accelerate energy convergence. It examines the flip of each spin separately in parallel, then randomly selects and updates one of the spins that can be flipped [51], [53], as shown in Fig. 4(b). The spin operator assumes the current spin state is flipped and then calculates the

energy variation with an introduced dynamic offset generated in the annealing controller. Each spin is said to be able to be flipped if the energy variation is larger than a random number, which indicates that flipping the spin state could decrease the energy. One of the spins that can be flipped is chosen at the end of each MC step.

To realize parallel spin update, the so-called parallel annealing (PA) utilizes a two-layer structure, including momentum annealing (MA) [54] and stochastic cellular annealing (SCA) [18]. Each spin interacts with all replicas, but there is no interaction between spins or between replicas. The interaction between a spin and its replica, referred to as self-interaction, is computed in the annealing controller. It increases with time to ensure that the states of the spin and its replica are the same at the end of annealing. In one MC step, the spin operator simultaneously inspects all the spins and performs random update for each replica spin, as shown in Fig. 4(c). Finally, the flipped replica states are assigned to the corresponding spins for computation in the next MC step.

Compared to the CMOSA machine, DA and PA machines respectively require a dynamic offset generator and self-interaction generator in the annealing controller, as shown in Fig. 4. Although both DA and PA machines inspect all the spins in the spin operator, only the PA machine updates all spin states in each MC step. To enhance the solution quality, a replica exchange MC sampling method, known as parallel tempering (PT) has been considered for CMOSA and DA machines [55], [56]. It introduces replicas of spins and then updates them at different temperatures for annealing to traverse more local minima. A PA machine is vulnerable to being stuck at local minima. To address this issue, an exponentially decreasing temperature function with the dynamic offset is applied in [57], [58]. A ratio ranging from $1/N$ to 1 is used to limit the number of spins to be flipped in one MC step [59].

2) Dynamics-inspired Classical Ising Machines

Dynamics-inspired Ising machines can be classified into two types: one based on quantum annealing and the other using simulations of oscillator networks.

The first class is known as simulated quantum annealing (SQA) machines. SQA imitates quantum dynamics using an MC method [60], [61]. This quantum MC method simulates the quantum tunneling phenomena of a transverse-field Ising model, which uses multiple replicas of spins called Trotters. The transverse field in SQA plays a similar role as the temperature in SA to control the probability of transition between the states of spins. A general architecture is similar to the classical annealing machine. In the annealing controller, the transverse field decreases with MC steps, and random numbers are generated. In each Trotter, an energy variation is computed in series for each spin operator. The spin states are then updated in a probabilistic manner determined by the energy variation and random numbers. The computation starts from the first Trotter and the spin states in the same Trotter are computed sequentially. Current studies mainly focus on runtime acceleration by using temporal [62]–[64] or spatial parallelism [62], [65].

Based on the simulations of oscillator networks, the second class digitally models the dynamics of a network of Kerr-nonlinear parametric oscillators using simulated bifurcation

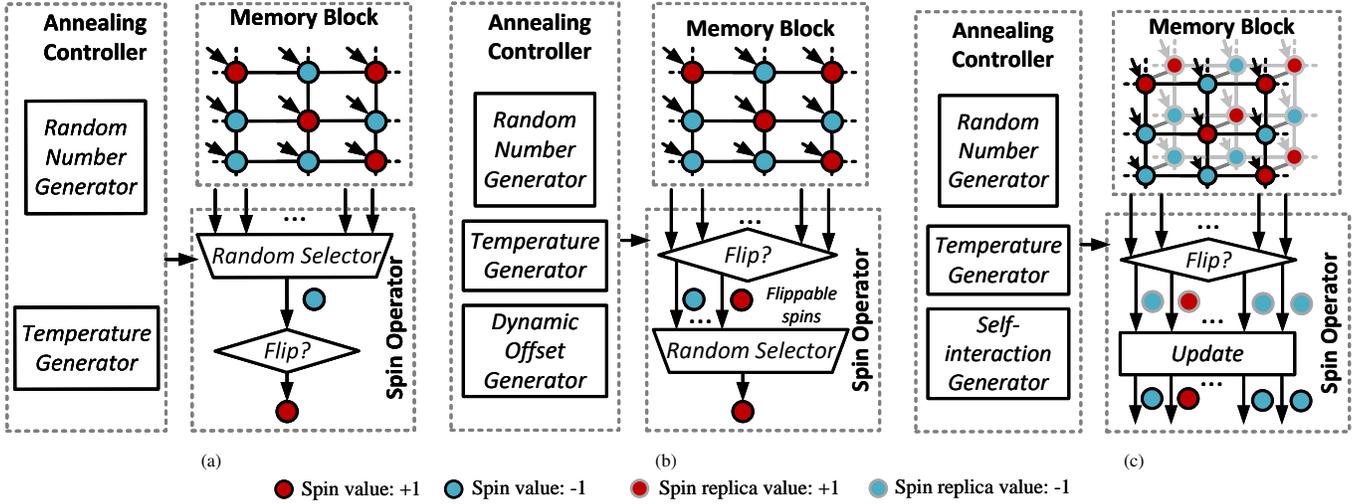


Fig. 4. Annealing Ising machines. (a) The CMOSA machine [50], (b) The DA machine [51], and (c) The PA machine [18].

(SB) algorithm [66]–[71], the degenerate optical parametric oscillators using emulated coherent Ising machines (ECIMs) [72], [73], or the electronic nonlinear oscillators using emulated oscillator Ising machines (EOIMs) [74], [75]. The discrete spin state is given by a continuous spin variable related to the oscillator position or phase. The signs of spin variables indicate the spin states at the end of a search. This type of Ising machines can be seen as an ordinary differential equation solver of the spin variables with respect to time using the Euler integration method. It is generally constructed by an evolution controller for computing the time-changing parameters, spin operators for evolving the spin variables with time, and memory blocks for spin variables and coupling coefficients. All the spin variables can be updated in parallel. For an ECIM design, the evolution controller also generates random numbers that follow a Gaussian distribution as noise to the system. However, the SB and EOIM designs do not rely on randomness. Thus, time evolutions of the spin variables in SB and EOIM are predictable since the computation is deterministic after random initialization. Compared to SB and ECIM designs that require many multiply-accumulate (MAC) units, the EOIM avoids massive multiplication operations. To improve hardware efficiency, ternary and logarithmic quantization of spin variables used in MACs are discussed in [69] for SB.

B. Classical Analog Oscillator based Ising Machines (OIMs)

Analog OIMs implemented using the conventional CMOS technology have raised a growing interest due to their advantage in such metrics as time-to-solution, energy-to-solution, and solution quality. The oscillators are arranged in a crossbar structure and the coupling circuits are implemented using resistors. Different types of oscillators have been considered, such as LC-tank [78], ring [36], [79], and Schmitt trigger oscillators [80]. In this section, we categorize the OIMs based on different implementations of the oscillators.

1) LC Oscillator-based Ising Machines (LC-OIMs)

The LC-OIM constructed with LC-tank oscillators was first proposed in [78]. It takes advantage of the self-annealing

properties of an oscillator network and its ability to simulate the Ising model with minor modifications. The spin state in the Ising model is represented by the oscillator phase. In order to convert the continuous oscillator phase into a bistable phase, a subharmonic injection locking (SHIL) signal is introduced. The coupling coefficients of the Ising model can be implemented through either resistive or capacitive connections between oscillators. The annealing process is simulated by increasing the SHIL signal’s amplitude. When oscillators naturally settle into a stable phase, the LC-OIM finds the (near-)ground state of a given Ising model, which provides a solution for the COP. Additionally, the LC-tank OIM has been expanded by using a printed circuit board (PCB) [38], which outperforms all classical annealing Ising machines by a significant margin.

2) Ring Oscillator-based Ising Machines (ROIMs)

LC-tank OIM implementations have shown great potential for solving COPs. However, embedding an LC oscillator into a chip has been a major challenge. Ahmed et al. [36], [79] have demonstrated a fully programmable OIM chip with a hexagonal topology by exploiting the high density and simplicity of ring oscillators. In addition, Moy et al. [81] have reported a larger ROIM chip that uses programmable transmission gates to couple ring oscillators for a higher precision to represent coupling coefficients.

3) Other Classical Analog Oscillator-based Ising Machines

To tackle specific challenges of the OIM, a range of oscillator designs have been developed. One such design is the Schmitt trigger OIM (ST-OIM) that eliminates the need of a SHIL system. Vaidya et al. [80] demonstrated that by incorporating an engineered feedback circuit into an electronic oscillator based on the Schmitt trigger design, it can generate the SHIL signal internally. This approach reduces the required die area and improves the scalability of the OIM chip. The differential OIM (DOIM) is another implementation of the OIM that achieves a coupling precision of 6 bits. Graber et al. [82] showed that a tunable differential oscillator design coupled with a digital-to-analog circuit can achieve a much wider number range for the coupling coefficients than any previous design, making OIMs more capable of solving complex COPs. Last but not the least,

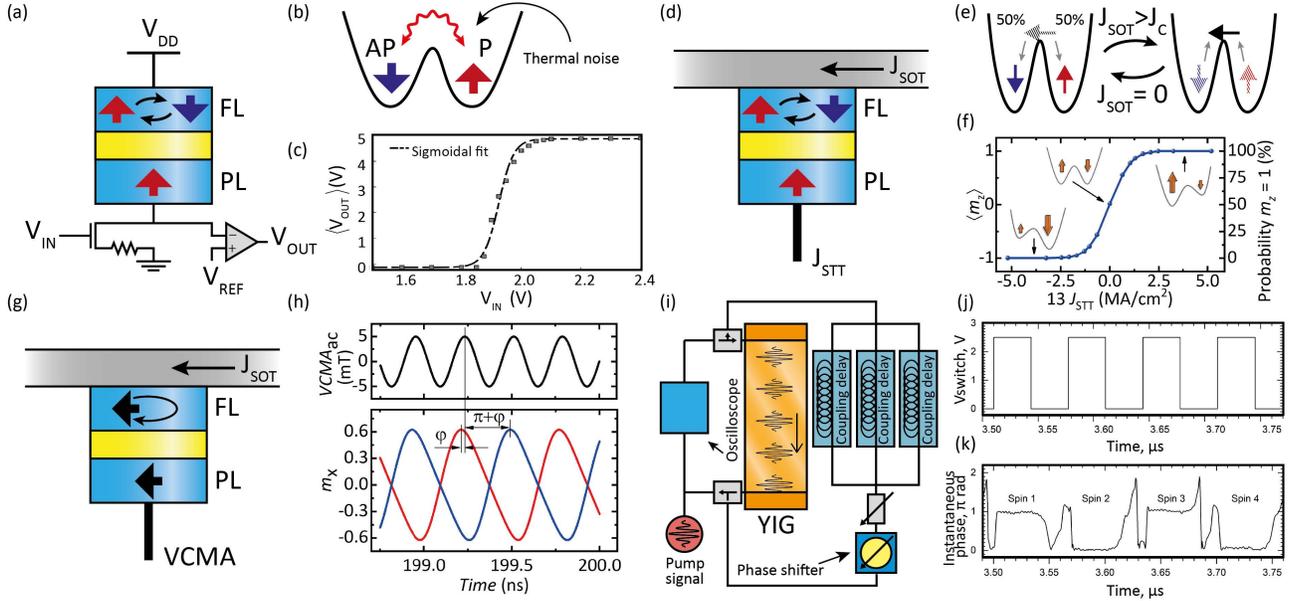


Fig. 5. Overview of SIMs. (a) sMTJ-PSIM diagram (PL: pinned layer; FL: free layer). The orientation of the FL magnetization changes the resistance and, thus, the measured V_{OUT} . (b) The FL magnetization switches between the two energetically stable configurations (anti-parallel and parallel) thanks to thermal noise. (c) The input voltage V_{IN} can tune the switching probability. The average of the V_{OUT} as a function of the V_{IN} is a sigmoid. Taken with permission from [76]. (d) 3TMTJ-PSIM diagram. (e) A J_{SOT} pulse can be used to force the FL magnetization in-plane and then out-of-plane again with a 50% probability. (f) This probability can be tuned using the J_{STT} . The average of the z-component of the FL magnetization is a sigmoid. (g) 3TMTJ-OSIM diagram. The J_{SOT} current drives the in-plane self-oscillations of the FL magnetization. (h) The VCMA effect can be used to force binarization of the oscillation phase thanks to injection locking. (i) SWIM diagram. Spin waves propagate through the YIG waveguide and interfere with each other due to the coupling delays in the line. (j) Control signal that forms oscillations into radio-frequency pulses. Taken with permission from [77]. (k) Instantaneous phase signal of the spin waves. Taken with permission from [77].

the tunable feature of the oscillator design is used to synchronize all oscillators to a single frequency, ensuring consistent and accurate results.

V. EMERGING CMOS COMPATIBLE ISING MACHINES

A. Spintronic Ising Machines (SIMs)

One of the most promising candidates for the hardware implementation of spins is the MTJ, thanks to its nanoscale size, low energy consumption, and CMOS compatibility [83]. MTJs are suitable for the implementation of several Ising machines' paradigms, such as probabilistic computing with p-bits [84], [19] and OIMs [17], as well as other computational paradigms [85], [86]. The high-speed dynamics and low power consumption of these devices potentially allow for a jump in the speed of solving some classes of COPs by several orders of magnitude [19] while also improving energy efficiency. In this section, we list the main existing designs for MTJ-based Ising machines. Each of these architectures can also be used in conjunction with techniques inherited from classical annealing Ising machines, such as DA, PA, SQA, and so on [61], [87]. However, the various tweaks necessary in order to implement the various schemes will not be discussed in this section.

1) Superparamagnetic MTJ-based Probabilistic Spintronic Ising Machines (sMTJ-PSIMs)

Probabilistic computing with p-bits can be used to implement an Ising machine paradigm that employs a tunable bistable stochastic unit as its spin [88], [89]. The p-bit continuously fluctuates in the time domain between the up and down configurations with a probability tuned by the states of its

topological neighboring p-bits. This can be implemented in hardware by creating a network of MTJs, where the FL's magnetization of each device represents an Ising spin [19], as shown schematically in Fig. 5(a). The change in the FL magnetization changes the resistance and thus the measured voltage V_{OUT} .

The MTJ is designed so that thermal fluctuations are strong enough to cause the magnetization to jump between the two stable configurations (see Fig. 5(b)). The tuning of the i^{th} p-bit is achieved by using an out-of-plane magnetic field or an input voltage adjusted to the state of the topological neighbors (the intensity is $\propto \sum J_{ij}m_j + h_i$). Fig. 5(c) shows the dependence of the average measured V_{OUT} on the V_{IN} . One advantage of this implementation is that the paradigm shifts from digitally discrete (software implementation of probabilistic computing) to intrinsically continuous dynamics. Thus, there is no need for sequential updates of directly coupled p-bits [90], and the input signal calculation can be performed in parallel [76], [19].

2) Three Terminal MTJ-based Probabilistic Spintronic Ising Machines (3TMTJ-PSIMs)

A tunable p-bit can be implemented with three terminal MTJs [17], combining current-induced spin-orbit-torque (SOT) [91] and spin-transfer-torque (STT) [92], as shown in Fig. 5(d). The dynamics of the FL magnetization of a perpendicular MTJ with circular cross-section are described by the Landau-Lifshitz-Gilbert-Slonczewski equation [93].

The MTJ is designed so that the energy landscape of the FL magnetization has two stable minima along the z-axis (Fig. 5(e)). After applying a large enough SOT current, the

FL magnetization is brought to a metastable state aligned along the direction of the spin-current (y-axis). When the SOT is switched off, the FL magnetization relaxes, with equal probability, towards one of the two z-axis directions [94]. To tune the switching probability an STT is applied in the third terminal, resulting in a sigmoidal behavior shown in Fig. 5(f), in which each point is obtained by averaging over 10^4 realizations with different seeds.

3) Three Terminal MTJ-based Oscillatory Spintronic Ising Machines (3TMTJ-OSIMs)

In 3TMTJ-OSIMs, the Ising spin is obtained with the phase binarization of the self-oscillations of the MTJ FL magnetization [17]. The MTJ is designed with an elliptical cross-section, an in-plane polarizer, and FL magnetization, as shown in Fig. 5(g). The self-oscillation state is achieved by applying a large enough SOT current [95]. The injection-locking signal that drives the binarization can be an AC voltage-controlled magnetic anisotropy (VCMA) effect or an AC STT current with double the self-oscillation frequency (Fig. 5(h)).

4) Spinwave Ising Machines (SWIMs)

SWIMs use artificial spin states obtained from the phase of spinwave RF pulses propagating through a medium [77]. Compared to paradigms such as CIMs, that use light and require kilometers of optical fiber in their experimental setups [12], with SWIMs it is possible to take advantage of the slow propagation speed of spinwaves to achieve exceptionally small setups, with waveguides that are only few centimeters long. Several wave packets are propagated along the Yttrium Iron Garnet (YIG) waveguide, with couplings being implemented through delay cables that allow pulses to interact with each other, as illustrated schematically in Fig. 5(i). The spinwaves are generated with the control signal shown in Fig. 5(j). By interacting with each other, the spinwaves oscillate with a binarized phase that encodes the solution of the problem, see Fig. 5(k).

B. Phase-Transition Material-based Ising Machines (PTMIM)

Phase-transition material (PTM) undergoes metal-insulator transitions under given electrical stimuli. That is, abrupt switching occurs from/to a high resistivity state (insulating phase) to/from a low resistivity state (metallic phase). Without electrical stimuli, it tends to stabilize in the insulating phase. When the applied voltage increases and the current density flowing through it reaches a given amount, an insulator-to-metal transition (IMT) occurs. Once in the metallic state, when the voltage decreases and the current density drops below a second given value, a metal-to-insulator transition (MIT) takes place. Fig. 6(a) shows the I-V characteristic of a generic PTM two-terminal device. The most widely used PTM is vanadium dioxide (VO_2). A compact oscillator has been proposed, as shown in Fig. 6(b) [96]- [97]. Fig. 6(c) depicts waveforms for the oscillator output. The state of the VO_2 is also shown to better illustrate the circuit behavior. $VO_{2,STATE}='INS'$ means the device is in the insulating state. $VO_{2,STATE}='MET'$ corresponds to the device in the metallic state. Assuming that the VO_2 is in an insulating state (marked with “A” in Fig. 6(c)), the oscillator output is discharged through the resistor. This increases the voltage drop across the VO_2 ($V_{DD}-V_{OUT}$) and so does the current through

it. Once enough current density circulates, it switches to the metallic state (marked with “B” in Fig. 6(c)). Equivalently, using the electrical model, switching to the metallic state occurs once the VO_2 voltage reaches V_{IMT} . The capacitor is then charged through the VO_2 . This charging is very fast because of the low R_{MET} value. The voltage seen by the VO_2 decreases until it reaches V_{MIT} and the transition from metal to insulator state occurs. These nano-oscillators are attractive due to their area and potential energy efficiency.

Coupled oscillators implemented with VO_2 devices have been employed for pattern recognition applications, as reported in [98] and [99]. However, one of the applications that is currently attracting the most interest is the OIM [100]- [101]. As in previously described OIMs, SHIL is a requirement for being able to discretize oscillator phases into two possible values. Some works on PTMIMs explore the impact of the SHIL scheme on the performance of the OIMs [100]–[103]. In particular, an experimental implementation of an Ising machine using coupled phase-transition nano-oscillators with VO_2 devices has been reported [102]. This system solves the MCP of a 4-node unweighted fully connected graph with a success probability of 96.7% over 300 trials. This work establishes a link between the natural stochasticity of the oscillators, the amplitude of the SHIL signal and classical annealing. In [103], by analyzing the performance of a network of coupled stochastic injection-locked VO_2 oscillators in terms of its Lyapunov energy and its continuous-time dynamics, how the dynamics can be harnessed to perform classical annealing is presented.

In [100], experimental results with eight coupled VO_2 oscillators are reported. The system is configured to solve an MCP for an unweighted Möbius Ladder graph, and delves into the relationship between the shape of the SHIL and the previously mentioned annealing mechanism. A SHIL signal of increasing amplitude is explored. It shows that the success probability can be improved from 30% to 96% when the SHIL signal is smoothly increased to its maximum value for 600 oscillation cycles, instead of using a constant SHIL amplitude. The rationale behind this is that it exploits noise (including the stochasticity of VO_2) to escape from local energy minima. In [101], different factors that impact the probability of the system reaching the ground state of the Ising Hamiltonian and, therefore, the optimum solution to the corresponding optimization problem, are analyzed by electrical simulation. The initial phase relationship of the oscillators, noise, and coupling strength is considered in addition to the SHIL amplitude. Experimentally calibrated numerical simulations of larger dense systems show the high energy efficiency of 3.3×10^7 solutions/sec/Watt for 100-node dense MCPs. They exhibit $5\times$ improvement over a recently demonstrated memristor-based Hopfield network and several orders of magnitude improvement over other candidates such as central processing units (CPUs) and graphics processing units (GPUs), quantum annealer and photonic Ising machine approaches [100]. In [104], an experimental demonstration with nine capacitively-coupled VO_2 oscillators is reported for solving MCP and MAX-3-SAT problems.

Capacitive coupling plays the role of anti-ferromagnetism interactions in the Ising model. Ferromagnetism interactions require resistive coupling. A differential VO_2 oscillator (Fig. 6(d))

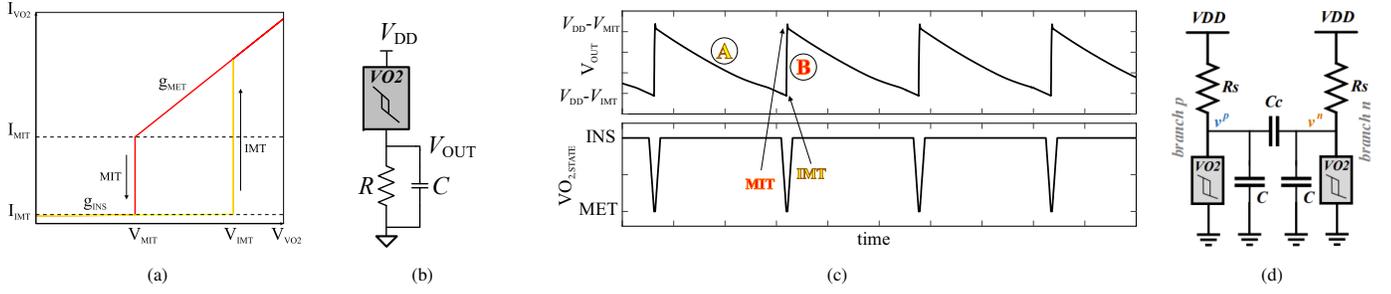


Fig. 6. (a) Generic I-V curve of a PTM device. The PTM oscillator: (b) circuit topology, and (c) operation waveforms in which points corresponding to insulator-to-metal and metal-to-insulator transitions are marked with arrows. (d) Differential VO_2 oscillator.

has been proposed [105], [106], which resorts to a capacitive coupling to force both outputs to be in anti-phase (180° phase difference). The differential oscillator in Fig. 6(d) allows implementing both types of interactions using only capacitive or resistive coupling. This is very attractive from the point of view of implementing coupling elements with memristors or ferroelectrics devices in crossbar architectures. Additionally, device mismatch has less impact on these differential structures than their single-ended counterparts [106].

VI. DISCUSSION AND EMERGING APPLICATIONS

Table I shows the main features of some state-of-the-art Ising machines. It is important to note that a quantitative comparison of the collected solutions is not straightforward since they differ significantly in the number of spins (between 4 and $147k$), the architecture used and the problem solved. Moreover, the measurements in accuracy and time-to-solution heavily depend on the benchmarks used and their size and topology, which is not shown in Table I.

Significant progress has been made for CMOS-based solutions. The heuristic algorithms for CMOS digital Ising machines can also be implemented on general-purpose hardware, such as CPUs and GPUs [62], [135]. The study on CMOS-based solutions has evolved to multi-chip design for improving the scalability of an Ising machine [136]. Moreover, most of COPs are formulated using the LIM with an arbitrary sparsely-connected topology to save hardware and reduce memory overheads, so using programmable spin connections becomes interesting [108]. Some emerging techniques, such as approximate computing [18], [55], [58], [128], stochastic computing [137], [138], in memory computing [139]–[144], and machine learning [145]–[147] have recently shown great potential for low power and high speed.

Ising machines based on the synchronization of coupled oscillators exhibit advantages in terms of computation speed. This is due to the parallel or collective computing on which they are based. Pure CMOS solutions have been demonstrated with a much larger number of spins than those based on emerging devices such as phase transition materials. However, the latter implementations show great potential in terms of power per spin, especially considering that the reported results are for an immature technology whose performance is expected to be improved.

Table II summarizes emerging applications of Ising machines. They include vehicle routing [114], [115], [148], control

parameter optimization for robotics [116], and wind farm layout optimization [117]. In finance, the Ising machine facilitates portfolio management [118] and fraud detection [119], while in production, it helps reduce worker travel distance and required workforce [120], [121]. The Ising machine also excels in optimizing communication systems [124], [125], accelerating the signal decoding in multi-input multi-output systems [123], and accelerating network migration [149]. In quantum computing, it aids in encryption key exchange between quantum network nodes [126] and error correction [127]. It also shows promise in improving image segmentation quality [129], optimizing circuits and PCB layouts [130], [133], [135], [150], and molecule structure-based drug discovery [53].

VII. SUMMARY, CHALLENGES AND PROSPECTS

This article provides a systematic review of the recent findings on (1) Ising formulations of COPs, (2) embedding an arbitrary Ising problem to the topology of an Ising machine, and (3) CMOS and CMOS-compatible Ising machines.

For COPs not easily encoded as QUBO instances, the efficiency of using different formulations has a profound influence on subsequent processing. The formulation strategies can be characterized by the number of required spins and accuracy, and there is no trivial way to identify the optimal approach for a given COP. Therefore, the efficiency of formulation strategies for a specific type of COP would be interesting to investigate.

For a CMOS Ising machine, the current approaches of updating spin states in parallel result in significantly increased hardware overhead. There is currently no suggestion that an energy minimization schedule optimal for any topology exists. As some problems require hard constraints while others do not, or as some only have one exact solution while others have many ground states, the optimal energy minimization schedule changes. Moreover, one application of the Ising machine may require high-quality approximate solutions in a very short time while another may need to find the absolute ground state in the shortest time possible. In any case, additional research to gather further insights is required.

CMOS compatible Ising machines have remarkable potential due to their high speed and energy efficiency, but they face additional challenges due to the peculiarities of the device used. For all categories of SIMs, perfecting the manufacturing of the MTJ or the YIG (in the case of SWIMs) is fundamental to have consistent physical properties to implement the Ising spin. sMTJ-PSIMs face the additional challenge of memory retention,

TABLE I
A COMPARISON OF STATE-OF-THE-ART ISING MACHINES

Ising Machines	Comput. Para./ Heuris. Alg.	Platform/ Technology	# of Spin	Topology	Coupling Precision	Coupling Type	Spin Type	Power per Spin	Area per Spin	COP
D-Wave'11 [107]	QA	Superconductor	2,048	Chimera	N/A	Qubit flux	Qubit	25 kW	N/A	N/A
Sci.Adv.'21 [12]	CC	Optics+FPGA	100k	Complete	3 levels	Coherent light	DOPO pulse	N/A	N/A	MCP
JSSCC'16 [37]	CMOSA	CMOS 65 nm	20k	3D Lattice	2 bit	SRAM	SRAM	49 mW	270 μm^2	MCP
ISSCC'21 [52]	CMOSA	CMOS 40 nm	147k ^a	King	5 bits	On-chip mem.	Register	N/A	552 μm^2	MCP
JSSC'24 [108]	CMOSA	CMOS 65 nm	1024	Lattice ^b	8 bits	SRAM	Register	1.14 μW	330 μm^2	MCP/SAT
JSSCC'23 [109]	CMOSA	CMOS 65 nm	6.4k	King	2 bits	SRAM	DRAM	0.11 μW	48 μm^2	MCP
Fujitsu'21 [110]	DA	Hybrid ^c	100k	Complete	64 bits	N/A	N/A	N/A	N/A	QAP
Phys.Rev.E'19 [54]	MA	4 GPUs	100k	Complete	10 bits	Proc. mem.	Proc. mem.	N/A	N/A	MCP
JSSC'20 [18]	SCA	CMOS 65 nm	512	Complete	5 bits	SRAM	Register	1.29 mW	N/A	MCP
ISSCC'23 [59]	Dynamic ^d	CMOS 40 nm	2048	Complete	8 bits	SRAM	Register	14-46 μW	722 μm^2	MCP
TETC'19 [65]	SQA	2 FPGAs +CPU	32, 768	Complete	32 bits ^e	Proc. mem.	Proc. mem.	N/A	N/A	NPP
Nat.Electron.'21 [111]	SB	8 FPGAs	16384	Complete	N/A	On-chip mem.	On-chip mem.	N/A	N/A	MCP
Nat.Electron.'22 [81]	Sync.+SHIL	CMOS 65 nm	1968	King	5 levels	TG coupling	Osc. phase	21.3 μW	< 53 μm^2	MCP
Nat.Electron.'23 [112]	Sync.+SHIL	CMOS 65 nm	48	Complete	29 levels	TG coupling	Osc. phase	2.3 mW	N/A	MCP
JSSC'21 [36]	Sync.+SHIL	CMOS 65 nm	560	Hexagonal	1 bit	Latch-based coup.	Osc. phase	41 μW	946 μm^2	MCP
Nat.Comput.'21 [113]	Sync.+SHIL	PCB	240	Chimera	3 levels	Potentiometer	Osc. phase	20.8 mW	N/A	MCP
Commun.Phys.'23 [77]	Sync.+SHIL	YIG+FPGA	8	Nearest	1 bit	Delay coupling	Osc. phase	2 W	70 μm^2	MCP
Nat.'19 [76]	Sync.+SHIL	MTJs	8	Complete	12 bit	On-chip mem.	Mag. dir.	200 μW	N/A	Fact.
Nat.Electron.'21 [100]	Sync.+SHIL	PTM Simulation	100	Complete	1 bit	Capac.	Osc. phase	12 μW	N/A	MCP
Nat.Comm.'24 [104]	Sync.+SHIL	PTM	9	Complete	1 bit	Capac.	Osc. phase	180 μW	N/A	MCP/SAT

Comput. Para./Heuris. Alg. (Computing Paradigms/Heuristic Algorithms): QA (Quantum annealing), CC (Coherent computing), SA (Simulated annealing), CMOSA (CMOS annealing), DA (Digital annealing), MA (Momentum annealing), SCA (Stochastic cellular annealing), SQA (Simulated quantum annealing), SB (Simulated bifurcation), Sync. + SHIL (Synchronization of coupled oscillators using SHIL); #: the number; Coeff. Prec.: coefficient precision; Acc.: accuracy; TG: Transmission-gate coupling; a: It is a 9-chip design with an eight-way chip-to-chip connection; b: The spin connectivity is programmable (4 spins with 28 interactions); c: The system uses a hybrid software and hardware implementation; d: The computing method is dynamically configured using CMOSA or SCA; e: Floating point number.

TABLE II
EMERGING APPLICATIONS OF ISING MACHINES

Scenarios	Application Fields	Accuracy
Smart City	Transportation and vehicles [114], [115]	M
	Robotics [116]	M
	Energy system [117]	M
Finance	Portfolio and trading [118]	H
	Fraud detection [119]	L
Industrial Production	Warehouse assignment [120], [121]	M
	Shift scheduling [121], [122]	L
Communication Networks	MIMO detection [123]	M
	Resource allocation [124], [125]	M
Quantum Computation	Quantum key distribution [126]	M
	Quantum error correction [127]	M
Medical, Chemical and Pharmaceutical applications	Medical CT imaging [128], [129]	M
	Drug discovery [53]	H
Materials and Devices	Materials informatics [130]	M
	Magnetic devices [131], [132]	M
	VLSI designs [133]–[135]	M

'H', 'M', and 'L' indicate the accuracy requirement is high, moderate, or low.

and, in all MTJ-based paradigms, the way the coupling between spins is computed, changes the overall design drastically. If it is waved in CMOS, the main issue becomes managing the connection between the MTJs and the CMOS part. If it is analogically computed, the issue becomes designing a robust strategy to physically perform the MAC operation necessary for the update. In SWIMs, managing complex topologies as the number of spins increases is also an important issue to address.

PTMIMs-based oscillatory neural networks using VO_2 devices for solving COPs hold great interest, driven by their potential for fast and energy-efficient problem resolution. Fabricating VO_2 devices according to semiconductor industry standards

allows for seamless integration at the back-end-of-line, ensuring compatibility with CMOS technology. Recently, devices fabricated on a silicon platform with a hafnium oxide interlayer have been demonstrated [104]. The dynamics of coupled oscillators allow for achieving a solution in reduced time due to the parallel and continuous-time processing of the phases. Furthermore, the combined exploitation of the variability of VO_2 devices together with subharmonic injection locking enables a very powerful mechanism to maximize the success rate in reaching the optimal solution. While the prospects are promising, there are still open questions concerning the scaling of such systems, especially regarding the adoption of sparse interconnection schemes.

ACKNOWLEDGMENT

The work at the University of Alberta was supported by the Natural Sciences and Engineering Research Council of Canada (No. RES0048688, RES0051374 and RES0054326). The work at the University of Messina was funded by the Italian Ministry of University and Research (No. PRIN 2020LWPKH7). The work at the Instituto de Microelectrónica de Sevilla IMSE-CNM was funded by the NeurONN Project (Horizon 2020 - Grant agreement ID: 871501).

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